



FEZA GÜRSEY
FİZİK VE MATEMATİK
UYGULAMA ve
ARAŞTIRMA MERKEZİ

2nd FGC-IPM Joint Mini Workshop “Arithmetic of Local and Global Fields”



پژوهشگاه دانش‌های بنیادی (مرکز تحقیقات فیزیک نظری و ریاضیات)
IPM
INSTITUTE FOR RESEARCH IN FUNDAMENTAL SCIENCES

Feza Gürsey Center for Physics and Mathematics
Boğaziçi University-Kandilli Campus
Üsküdar 34684, İstanbul-TURKEY

September 26 and 28, 2022

Location: Work has been underway since early Summer to renovate the building of FGC. Therefore, the workshop talks will take place in the Mathematics Department of Boğaziçi University, 34342 Bebek, Istanbul /Turkey.

Zoom links: T.B.A.

Speakers: Abbas Maarefparvar (IPM-Tehran); K. İlhan İkedâ (FGC-Istanbul).

Speaker: Abbas Maarefparvar (IPM-Tehran)

Title: On Pólya groups of algebraic number fields: Some applications and generalizations.

Dates: Sept. 26, 2022, Monday 10:30-12:00 and Sept. 28, 2022, Wednesday 09:00-10:30

Abstract:

Pólya groups are subgroups of ideal class groups of number fields generated by all classes of Bhargava generalized factorial ideals, which more classically, can also be defined in terms of integer valued polynomials. Considering the action of Galois groups on class groups of Galois number fields, one can show that Pólya groups coincide with strongly ambiguous ideal class groups, and investigating on this subject can be seen as a generalization of Gauss's genus theory as Hilbert did for quadratic number fields. In this talk, I will present some well known results on Pólya groups of number fields in various degrees, and introduce some generalizations of this notion to relative case, namely the relative Pólya groups and Ostrowski quotients for finite extensions of number fields. In particular, I will talk about the “BRZ”, a four-term exact sequence obtained from some cohomological results of Brumer-Rosen and Zantema, and give some its applications. If time permits, I will talk also about the recent attempts for finding some analogies in elliptic curves.

Reference List:

1. J. L. Chabert, “From Pólya fields to Pólya groups (I) Galois extensions.” J. Number Theory, Vol. **203**, (2019), 360-375.
2. J. L. Chabert, E. Halberstadt, “From Pólya fields to Pólya groups (II): non-Galois number fields.” J. Number Theory. Vol. **220**, (2021), 295-319.
3. A. Leriche, “Cubic, quartic and sextic Pólya fields.” J. Number Theory, Vol. **133**, (2013), 59-71.
4. A. Leriche, “About the embedding of a number field in a Pólya field.” J. Number Theory, Vol. **45**, (2014), 210-229.
5. A. Maarefparvar, A. Rajaei, “Pólya S_3 -extensions of \mathbb{Q} .” Proc. Roy. Soc. Edinburgh Sect. A, Vol. **149**, (2019), 1421-1433.
6. A. Maarefparvar, A. Rajaei, “Relative Pólya group and Pólya dihedral extensions of \mathbb{Q} .” J. Number Theory, Vol. **207**, (2020), 367-384.
7. A. Maarefparvar, “The analogue of the BRZ exact sequence for Tate-Shafarevich Groups”. (2022), Submitted. \url{<https://arxiv.org/abs/2202.04922>}.
8. E. Shahoseini, A. Rajaei, A. Maarefparvar, “Ostrowski quotients for finite extensions of number fields.” To appear in Pacific Journal of Mathematics, (2021), \url{<https://arxiv.org/abs/2111.00442>}.
9. H. Zantema, “Integer valued polynomials over a number field.” Manuscripta Math., Vol. **40**, (1982), 155-203.

Speaker: K. İlhan İkedâ (FGC-İstanbul)

Title: *Ono reciprocity law, absolute arithmetic and F_1 -geometry**

Date: September 28, 2022, Wednesday 11:00-12:30.

Abstract:

In this seminar talk we first introduce the reciprocity law of Ono for finite Galois extensions K/k of a global field k , which is a non-abelian generalization of the global Artin reciprocity law for finite abelian extensions E/k of k . We then plan to discuss further “functorial” properties of the Ono reciprocity law for K/k and, “passing to limits”, get the Ono reciprocity law for k^{sep}/k . Finally, we plan to discuss the relationship between this theory with the absolute arithmetic and F_1 -geometry.

* Joint work with Serkan Kızılavuz (Eskişehir Technical University).

Reference List:

1. S. Lang, “*Algebraic Number Theory (Second Edition)*”, GTM **110**, Springer-Verlag, Berlin and New York, 1994.
2. T. Ono, “*A note on the Artin map*”, Proc. Japan Acad., **65**, Ser. A, 1989, 304—306.
3. T. Ono, “*A note on the Artin map II*”, Proc. Japan Acad., **66**, Ser. A, 1990, 132—136.
4. T. Ono, “*A note on the Artin map III*”, Proc. Japan Acad., **67**, Ser. A, 1991, 79—81.
5. M. Rosen, “*Number Theory in Function Fields*”, GTM **210**, Springer-Verlag, Berlin and New York, 2002.
6. J-P Serre, “*Linear Representations of Finite Groups*”, GTM **42**, Springer-Verlag, Berlin and New York, 1977.
7. C. Thas, “*Absolute Arithmetic and F_1 -geometry*”. E.M.S Publ. House, Zurich, 2016.

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